Pennsylvania Department of Education
Transfer and Articulation Oversight Committee
Approved by TAOC on November 22, 2010
Amended April 11, 2012

PENNSYLVANIA STATEWIDE PROGRAM-TO-PROGRAM ARTICULATION AGREEMENT IN MATHEMATICS

Overview

In accordance with Act 50 of 2009, institutions participating in Pennsylvania's statewide college credit transfer system agree to the following policies governing the transfer of credits from a participating associate-degree granting institution into a participating four-year college or university. This agreement specifically ensures that a student who successfully completes an Associate of Arts (AA) or Associate of Science (AS) degree in Mathematics or any AA or AS degree that incorporates the required competencies at a participating institution can transfer the full degree into a parallel bachelor degree program in Mathematics at a participating four-year institution.

In order for students to transfer the full associate degree into a parallel bachelor degree program at a participating four-year institution, all of the following criteria must be met:

- Successful completion of an associate degree that includes all of the required major competencies identified in this Agreement.
- Successful completion of at least 30 credits of foundation courses from the Transfer Credit Framework.

See Appendix A: Program-to-Program Articulation Model for Mathematics.

It is therefore understood that students meeting these requirements will be considered by both the associate degree granting institution and the receiving four-year institution to possess the knowledge, skills and abilities necessary for entry as a junior into a parallel bachelor degree program in Mathematics.

References to courses in all agreements designate competencies and are not to be construed as making a reference to a specific course at a specific institution. Course titles in the agreements are presented for guidance in advising students as to which coursework they should take even though the course at the student's college may not have the specific title mentioned in the agreement.[1]

REQUIRED Major-Specific Content Areas

Under this Agreement, a fully-transferable associate degree in the field of Mathematics must include competencies from two primary content areas:

1. **Calculus**
2. **Foundations of Higher Mathematics/Mathematical Proof**

Institutions may determine how the competencies identified in these primary content areas are met. For example, one institution may choose to embed the Calculus competencies in four 3-credit courses, while another institution teaches the same competencies in three 4-credit courses. How an institution incorporates the competencies into the associate degree program does not affect the transferability of the associate degree under this Agreement in so long as all of the competencies are met.

See Appendix A: Program-to-Program Articulation Model for Mathematics.

1. Calculus

The world around us is constantly changing. Calculus is the branch of mathematics that has been developed to study changes. Therefore, the competencies acquired through the successful study of calculus provide mathematicians, scientists and other professionals the tools for understanding the changes that occur in their disciplines which, in turn, enables them to solve problems in their disciplines.

The following competencies have been identified as essential for comparable preparation in this content area:

Competency 1: Utilize the concept of limit.

[1] Adopted by TAOC and added to the agreement on April 11, 2012.

Competency 2: Differentiate functions.
Competency 3: Use differential calculus to sketch curves and to solve applied problems.
Competency 4: Integrate functions by approximation and by use of anti-derivatives.
Competency 5: Use integral calculus to determine area and to solve applied problems.
Competency 6: Differentiate and integrate using transcendental functions.
Competency 7: Integrate functions using special methods.
Competency 8: Relate the functional and geometric properties of conic sections, curves given in parametric form, and polar curves.
Competency 9: Use vectors to solve 2-space and 3-space geometrical problems.
Competency 10: Use vector-valued functions to describe motion in space.
Competency 11: Find partial derivatives of functions of two or more variables.
Competency 12: Use partial differentiation to solve applied problems.
Competency 13: Evaluate multiple integrals.
Competency 14: Use multiple integrals to solve applied problems.
Competency 15: Use techniques of vector analysis.
Competency 16: Test infinite series for convergence or divergence.

See Appendix B: Competencies for Preparation in Calculus.

Historically, these competencies commonly have been found in courses such as Calculus I (4 credits), Calculus II (4 credits) and Calculus III (4 credits). However, each associate institution may determine the format in which the competencies are presented.

2. Foundations of Higher Mathematics/Mathematical Proof

Mathematical systems are built up logically from statements that are true. In a mathematical system, a statement that is not an axiom or a definition is not accepted as true until it has been proven. Therefore, the ability to prove mathematical theorems and to read and comprehend the proofs of mathematical theorems is a necessary competency for the understanding of the structure of mathematics.

The following competencies have been identified as essential for comparable preparation in this content area:

Competency 1: Use of Logic
Competency 2: Use of Quantifiers
Competency 3: Use of Set Theory
Competency 4: Proof Techniques
Competency 5: Relations and Functions
Competency 6: Applying Proof Techniques

See Appendix C: Competencies for Preparation in Foundations of Higher Mathematics/ Mathematical Proof.

RECOMMENDED Major-Specific Content Areas

In addition to the required competencies listed above, students transferring into a bachelor degree program in Mathematics would also benefit from acquiring competencies in at least ONE of the following content areas:

1. **Linear Algebra**
2. **Differential Equations**

Students will not be penalized for not completing competencies in one or both of these areas of study, though exposure to these additional mathematical principles would greatly benefit a Math major transferring at the junior level. See Appendix A: Program-to-Program Articulation Model for Mathematics.

1. Linear Algebra

Many phenomena in the world have an underlying structure which follows basic algebraic rules. One of these structures is the vector space and linear algebra is the area of mathematics that has been developed to model phenomena that satisfy this structure. Competencies acquired in the successful study of linear

algebra not only make it possible to study and understand the development of vector space models but also provide the foundation for the study of more advanced algebraic structures.

The following competencies have been identified as essential for comparable preparation in this content area:

Competency 1: Solving Systems of Linear Equations
Competency 2: Matrix Arithmetic
Competency 3: Determinants
Competency 4: Vector Spaces
Competency 5: Inner Product Spaces
Competency 6: Eigentheory
Competency 7: Linear Transformations

See Appendix D: Competencies for Preparation in Linear Algebra.

2. **Differential Equations**
Many real world phenomena can be modeled by differential equations. Competencies acquired in the successful study of differential equations will provide necessary preparation for students to study and understand more advanced topics in applied mathematics.

The following competencies have been identified as essential for comparable preparation in this content area:

Competency 1: Basic Concepts
Competency 2: First Order Ordinary Differential Equations
Competency 3: Second Order Ordinary Differential Equations
Competency 4: Series Solutions
Competency 5: Laplace Transforms

See Appendix E: Competencies for Preparation in Differential Equations.

RECOMMENDED General Coursework

Associate degree students transferring into a bachelor degree program in Mathematics would also benefit by completing coursework outside of the major. Specifically, the following coursework is recommended, but not required:

1. Calculus-based Physics – Recommend students complete one or two courses of Calculus-based Physics. Two courses are preferred. Courses may be used to satisfy the requirements of Category 4 of the Transfer Credit Framework for Math Majors.

2. Computer Science – Recommend students complete 1 or 2 courses in Computer Science in which the student learns to translate mathematical algorithms into a compiled computer language (e.g., Java, Introduction to C++ Programming, etc.) Two courses are preferred.

As with the Recommended Major-Specific Competencies, students will not be penalized for not completing the recommended general coursework. See Appendix A: Program-to-Program Articulation Model for Mathematics.

Transfer Credit Framework

Under Act 114 of 2006, the Commonwealth's statewide college credit transfer system includes an advising tool called the "Transfer Credit Framework". The Framework allows students to seamlessly transfer up to 30 credits of foundation courses from one participating college or university to another and have those courses count toward graduation. See Appendix F Transfer Credit Framework.

The Framework consists of a menu of 49 courses that fall within six broad categories: English, public speaking, math, science, fine arts and humanities, and the behavioral and social sciences. To fully benefit from the Framework, students are advised to select a range of courses according to the criteria for each category.

Under this Agreement, students may select courses according to the criteria indicated for Framework Categories 1, 2, 5 and 6. In Framework Category 3, students may apply a maximum of 8 credits (2 courses) completed as part of the Required Major-Specific Content Area of Calculus. Likewise, a maximum of 8 credits (2 courses) in Calculus-based Physics, recommended coursework outside of the field of Mathematics, may be used to satisfy the requirements of Framework Category 4. (See Appendix A: Program-to-Program Articulation Model for Mathematics.)

Students are advised to work with an advisor to select courses related to their associate degree program, transfer major and personal interests.

Appendix A: Program-to-Program Articulation Model for Mathematics

Major-Specific Content Areas – REQUIRED	**Transfer Criteria**
Calculus	
Foundations of Higher Mathematics/Mathematical Proof	
Major-Specific Content Areas – RECOMMENDED	**Transfer Criteria**
Students are strongly advised to acquire competencies in at least ONE of the following content areas: **1. Linear Algebra** **2. Differential Equations**	
RECOMMENDED Coursework Outside of the Discipline	**Transfer Criteria**
1. **At least one course of Calculus-based Physics, preferably two courses.** (Math majors may use this coursework to satisfy the requirements of Category 4 of the Transfer Credit Framework. See below.) 2. **At least one semester of Computer Science, preferably two courses.** (Students must be able to translate mathematical algorithms into a compiled computer language such as Java or C++.)	Student should consult an advisor before enrolling in the recommended coursework.
TRANSFER CREDIT FRAMEWORK	**Transfer Criteria**
Category 1	**1 course** to be selected by the student with the assistance of an advisor
Category 2	**1 course** to be selected by the student with the assistance of an advisor
Category 3	**Calculus I** **Calculus II** Math majors may use up to 8 credits of coursework from the Major-Specific Content Area of Calculus to satisfy Category 3 requirements.
Category 4	**RECOMMEND 1 or 2 courses of Calculus-based Physics** Math majors may use up to 8 credits of Calculus-based Physics to satisfy Category 4 requirements.
Category 5	**2 courses** to be selected by the student with the assistance of an advisor
Category 6	**2 courses** to be selected by the student with the assistance of an advisor

Appendix B: Competencies for Preparation in Calculus

Competency 1: Utilize the concept of limit.
Behavioral Objectives: In order to attain this competency, the student should be able to:
1.1 determine limits using a table of values or graph.
1.2 evaluate limits of polynomial, rational, and trigonometric functions by direct substitution.
1.3 where substitution yields an indeterminate form, find limits by cancellation and rationalization techniques or by the use of identities.
1.4 use L'Hopital's Rule to find limits of indeterminate forms.
1.5 evaluate limits using the Squeeze Theorem.
1.6 use limit theorems involving sums, differences, products, and quotients of functions.
1.7 indicate whether a function is continuous or discontinuous; if discontinuous, give all points of discontinuity.
1.8 determine limits at infinity.

Competency 2: Differentiate functions.
Behavioral Objectives: In order to attain this competency, the student should be able to:
2.1 define and interpret the derivative of a function.
2.2 compute derivatives of functions using the definition.
2.3 obtain the derivatives of sums, products, quotients, and powers of polynomial, trigonometric, and transcendental functions using the general formulas for differentiation.
2.4 use the chain rule to differentiate the composition of functions.
2.5 find differentials.
2.6 differentiate implicitly.
2.7 find higher order derivatives.
2.8 evaluate derivatives.

Competency 3: Use differential calculus to sketch curves and to solve applied problems.
Behavioral Objectives: In order to attain this competency, the student should be able to:
3.1 find the intervals on which a function is increasing or decreasing and the intervals on which a function is concave upward or concave downward.
3.2 determine relative minima, relative maxima, and points of inflection, if any, and sketch the graph of a function.
3.3 find the equations of lines tangent and normal to a curve at a given point.
3.4 find the point(s) on a curve where the tangent line has a given slope.
3.5 use differentials to approximate values of non-linear functions.
3.6 approximate a solution for an equation using Newton's Method.
3.7 given a position function, calculate the velocity and acceleration of a particle and analyze its motion.
3.8 apply Rolle's Theorem and the Mean Value Theorem to a function.
3.9 solve applied related rate problems.
3.10 solve applied maximum-minimum problems.
3.11 apply the Extreme Value Theorem to a function.

Competency 4: Integrate functions by approximation and by use of antiderivatives.
Behavioral Objectives: In order to attain this competency, the student should be able to:
4.1 define the indefinite and definite integral of a function.
4.2 find antiderivatives by using the power rule and substitution.
4.3 integrate algebraic and trigonometric functions.
4.4 determine the constant of integration given sufficient conditions.
4.5 use the Fundamental Theorem of Calculus to evaluate definite integrals.
4.6 approximate an integral by the Trapezoidal Rule or Simpson's Rule.
4.7 use the 2^{nd} Fundamental Theorem of Integral Calculus
4.8 express the limit of a Riemann sum as a definite integral.

Competency 5: Use integral calculus to determine area and to solve applied problems.

Behavioral Objectives: In order to attain this competency, the student should be able to:
5.1 find the area of a region bounded by the graphs of given equations.
5.2 determine the volume of a solid of revolution by the disc and washer methods or by the shell method.
5.3 find the length of a plane curve.
5.4 determine the area of the surface of revolution.
5.5 calculate various physical quantities such as amount of work done by a variable force over an interval, moments, centers of mass, centroids, fluid pressure and fluid force.
5.6 calculate the average value of a function and use the Mean-Value Theorem for Integrals

Competency 6: Differentiate and integrate using transcendental functions.
Behavioral Objectives: In order to attain this competency, the student should be able to:
6.1 find derivatives of functions involving the natural logarithmic function.
6.2 integrate rational functions whose antiderivatives are natural logarithmic functions.
6.3 find the derivative of an inverse function.
6.4 differentiate and integrate natural exponential functions.
6.5 differentiate and integrate exponential functions that have bases other than *e*.
6.6 solve growth and decay problems.
6.7 differentiate inverse trigonometric, hyperbolic, and inverse hyperbolic functions.
6.8 integrate functions yielding inverse trigonometric, hyperbolic or inverse hyperbolic functions.

Competency 7: Integrate functions using special methods.
Behavioral Objectives: In order to attain this competency, the student should be able to:
7.1 integrate by parts.
7.2 integrate powers of trigonometric functions.
7.3 integrate using trigonometric substitution.
7.4 integrate using partial fraction decomposition.
7.5 integrate using tables.
7.6 evaluate improper integrals.

Competency 8: Relate the functional and geometric properties of conic sections, curves given in parametric form, and polar curves.
Behavioral Objectives: In order to attain this competency, the student should be able to:
8.1 given the equation of a conic section, identify its parts (e.g., center, vertices, foci, axes, asymptotes, eccentricity, etc.) and graph it.
8.2 find the equation of a conic section (circle, parabola, ellipse, hyperbola) given sufficient information about its parts.
8.3 graph a curve given by a set of parametric equations.
8.4 find a set of parametric equations to represent a curve.
8.5 find the slope of a tangent line to a curve given by a set of parametric equations
8.6 find the arc length of a curve given by a set of parametric equations.
8.7 transform equations from polar coordinates to rectangular coordinates and vice-versa.
8.8 sketch common polar graphs.
8.9 determine the slope of a tangent line to a polar graph.
8.10 find the area of a region bounded by a polar graph and the arc length of a polar graph.

Competency 9: Use vectors to solve 2-space and 3-space geometrical problems.
Behavioral Objectives: In order to attain this competency, the student should be able to:
9.1 write a vector in component form or as a linear combination of standard unit vectors.
9.2 graph a given a vector, unitize it, and find its magnitude and direction.
9.3 add, subtract, and form scalar multiples of vectors.
9.4 calculate the dot (scalar) product of two vectors and use the dot product to find the angle between two vectors, the direction cosines of a vector, and the projection of one vector onto another.
9.5 calculate the cross product of two vectors and the triple scalar product of three vectors.
9.6 find equations of lines and planes in 3-space, given sufficient data.
9.7 identify and sketch planes, cylinders, and quadric surfaces, given their equations.
9.8 Convert between rectangular, cylindrical and spherical coordinates.

Competency 10: Use vector-valued functions to describe motion in space.
Behavioral Objectives: In order to attain this competency, the student should be able to:
10.1 extend the concepts of limit, continuity, differentiation, and integration to vector-valued functions.
10.2 graph vector-valued functions.
10.3 differentiate a displacement (position) vector to find the velocity and acceleration vectors and the speed at a point.
10.4 use vector-valued functions to analyze projectile motion.
10.5 for a given vector-valued function, find a unit tangent, a unit normal, and the tangential and normal components of acceleration.
10.6 find the arc length and the curvature of a space curve described by a vector-valued function.

Competency 11: Find partial derivatives of functions of two or more variables.
Behavioral Objectives: In order to attain this competency, the student should be able to:
11.1 find the first-order partial derivatives of functions.
11.2 find higher order partial derivatives.
11.3 use the chain rule for partial derivatives.
11.4 calculate the total differential

Competency 12: Use partial differentiation to solve applied problems.
Behavioral Objectives: In order to attain this competency, the student should be able to:
12.1 find the directional derivative.
12.2 find the equation of the tangent plane to a surface at a given point.
12.3 find the gradient of a function.
12.4 maximize or minimize functions of two independent variables.
12.5 apply Lagrange Multipliers to maximum – minimum problems.

Competency 13: Evaluate multiple integrals.
Behavioral Objectives: In order to attain this competency, the student should be able to:
13.1 evaluate double integrals.
13.2 evaluate double integrals by use of polar coordinates.
13.3 evaluate triple integrals.
13.4 evaluate triple integrals by use of cylindrical coordinates.
13.5 evaluate triple integrals by use of spherical coordinates.

Competency 14: Use multiple integrals to solve applied problems.
Behavioral Objectives: In order to attain this competency, the student should be able to:
14.1 find areas by use of double integration.
14.2 locate the center of gravity and centroid of a solid.
14.3 find volumes by use of multiple integrals.
14.4 evaluate triple integrals to solve applied problems.
14.5 find surface area.

Competency 15: Use techniques of vector analysis.
Behavioral Objectives: In order to attain this competency, the student should be able to:
15.1 evaluate surface integrals.
15.2 evaluate line integrals.
15.3 find work done in a vector field.
15.4 determine the path-independent line integrals.
15.5 use Green's Theorem to compute line integrals or double integrals.
15.6 use the Divergence Theorem to compute surface integrals or triple integrals.
15.7 use Stokes' Theorem to compute line integrals or surface integrals.

Competency 16: Test infinite series for convergence or divergence.
Behavioral Objectives: In order to attain this competency, the student should be able to:

16.1 determine whether a sequence converges or diverges.
16.2 find the limit of convergent sequences.
16.3 determine whether a given geometric series or p-series converges or diverges.
16.4 find closed expressions for the sum of terms of an infinite geometric and telescoping series.
16.5 test for convergence or divergence of an infinite series of non-negative terms using, (a) direct comparison and limit comparison tests, (b) the integral test, (c) the ratio test, (d) the root test.
16.6 test for absolute convergence and conditional convergence of alternating series.
16.7 express functions as power series.
16.8 find the interval of convergence for power series.
16.9 write Maclaurin series expansions.
16.10 write Taylor series expansions.
16.11 compute using series expansions.
16.12 differentiate and integrate power series.
16.13 use the Remainder Term in Taylor's Theorem to perform error estimates

Appendix C: Competencies for Preparation in Foundations of Mathematics/ Mathematical Proof

Competency 1: Use of logic
Behavioral Objectives: In order to attain this competency, the student should be able to:
1.1 distinguish between statements in logic and non-statements in logic and determine the truth values of statements when possible
1.2 define the logical connectives (negation, conjunction, disjunction, conditional, biconditional) by the use of truth tables
1.3 construct truth tables for compound statements
1.4 use truth tables to determine tautologies and contradictions
1.5 use truth tables to determine the logical equivalence of two statements and the logical implication of two statements
1.6 identify the hypothesis of a conditional statement and the conclusion of a conditional statement
1.7 determine the converse, contrapositive, and inverse of a conditional statement
1.8 separate a biconditional into its two conditional statements
1.9 establish, recognize and use the basic algebraic properties of logical connectives
1.10 use the logic notation correctly

Competency 2: Use of Quantifiers
Behavioral Objectives: In order to attain this competency, the student should be able to:
2.1 recognize and use the universal quantifier and the existential quantifier and distinguish between them
2.2 negate quantified statements
2.3 determine the truth values of quantified statements, including those using combinations of quantifiers

Competency 3: Use of Set Theory
Behavioral Objectives: In order to attain this competency, the student should be able to:
3.1 distinguish between sets and elements
3.2 recognize the empty set and use its notation properly
3.3 define the basic set operations of complement, intersection and union in terms of the corresponding logical connectives and use the notation of set theory correctly
3.4 define and recognize subsets and set equality
3.4 establish, recognize and use the basic algebraic properties of sets
3.5 use the basic algebraic properties of sets to establish more set properties
3.6 determine intersections and unions of indexed families of sets
3.7 recognize and construct partitions of sets
3.8 distinguish among the basic sets of numbers: N, Z, Q, R, C

Competency 4: Proof Techniques
Behavioral Objectives: In order to attain this competency, the student should be able to:
4.1 recognize the format of a mathematical system in terms of axioms, definitions, and theorems
4.2 determine the validity of arguments and recognize valid consequences
4.3 properly use the rules of inference and identify fallacies
4.4 establish valid methods of proof using truth tables or logical equivalences including the method of direct proof, the method of proof by contrapositive, the method of proof by contradiction (indirect proof), the method of proof by cases, and the Principle of Mathematical Induction and its various forms
4.5 construct counterexamples for false statements

Competency 5: Relations and Functions
Behavioral Objectives: In order to attain this competency, the student should be able to:
5.1 define the Cartesian product of two sets and recognize that the elements are ordered pairs
5.2 define a relation from one set to another as a subset of a Cartesian product
5.3 determine the inverse of a relation
5.4 define properties of relations on a set such as reflexive, symmetric, antisymmetric and transitive
5.5 recognize equivalence relations and determine corresponding equivalence classes

5.6 recognize partial orders
5.7 define a function as a relation in which no first coordinate is used more than once
5.8 determine images and pre-images for functions
5.9 define the function properties of one-to-one and onto
5.10 define and recognize bijections and determine inverses of bijections
5.11 determine the intersection, union and composition of relations and their properties
5.12 determine the sum, difference and composition of functions and their properties

Competency 6: Applying Proof Techniques
Behavioral Objectives: In order to attain this competency, the student should be able to:
6.1 apply appropriate proof techniques to prove properties of sets including when one set is a subset of another set and when two sets are equal
6.2 apply appropriate proof techniques to prove properties about relations
6.3 apply appropriate proof techniques to prove properties about functions
6.4 apply appropriate proof techniques to prove properties about the natural numbers and the integers including divisibility properties and properties of congruence modulo n
6.5 apply appropriate proof techniques to prove basic properties involving real numbers
6.6 apply appropriate proof techniques to prove basic properties involving cardinality of sets

Appendix D: Competencies for Preparation in Linear Algebra

Competency 1: Solving Systems of Linear Equations
Behavioral Objectives: In order to attain this competency, the student should be able to:
1.1 Distinguish between homogeneous and nonhomogeneous linear systems
1.2 Identify consistent and inconsistent linear systems
1.3 Solve linear systems using Gaussian elimination and Gauss-Jordan elimination
1.4 Distinguish between consistent linear systems that have a unique solution and those that have infinitely many solutions and write these solutions using parameters
1.5 Rewrite linear systems as matrix equations and solve consistent n x n linear systems with unique solutions using the inverse method and Cramer's Rule where reasonable

Competency 2: Matrix Arithmetic
Behavioral Objectives: In order to attain this competency, the student should be able to:
2.1 Determine when two matrices are equal
2.2 Add and subtract matrices of the same size
2.3 Recognize when matrix multiplication is defined and multiply matrices under these conditions
2.4 Perform the operation of scalar multiplication on matrices
2.5 Determine the transpose of a square matrix
2.5 Determine the inverse of an invertible square matrix by using elementary row operations and the adjoint formula
2.6 Identify, prove and use the properties of matrix arithmetic
2.7 Recognize, define and prove properties about diagonal, triangular, and symmetric matrices

Competency 3: Determinants
Behavioral Objectives: In order to attain this competency, the student should be able to:
3.1 Define the determinant of a square matrix recursively by the method of cofactor expansion
3.2 Evaluate determinants using elementary row operations
3.3 Recognize and prove basic properties about determinants

Competency 4: Vector Spaces
Behavioral Objectives: In order to attain this competency, the student should be able to:
4.1 Recognize, define and give examples of real vector spaces including R^n
4.2 Define, recognize and prove basic results about subspaces of vector spaces
4.3 Determine if a finite set of vectors is linearly independent or linearly dependent
4.4 Determine if a finite set of vectors spans or does not span a vector space
4.5 Determine if a finite set of vectors forms a basis for a finite dimensional vector space
4.6 Determine a basis for a given subspace of a finite dimensional vector space
4.7 Relate matrices to the concepts of linear independence, spanning and basis
4.8 Determine the row space, null space and column space of a matrix and determine its rank and nullity and use the Dimension Theorem for Matrices
4.9 Use matrices to change from one basis to another in a finite dimensional vector space

Competency 5: Inner Product Spaces
Behavioral Objectives: In order to attain this competency, the student should be able to:
5.1 Recognize, define and give examples of real inner product spaces including R^n
5.2 Use the definition of inner product to define norm and distance in an inner product space
5.3 Define and prove basic results about orthogonality in an inner product space
5.4 Use the Gram-Schmidt Process to create an orthonormal basis for an inner product space
5.5 Define and use orthogonal matrices and prove basic properties about them

Competency 6: Eigentheory
Behavioral Objectives: In order to attain this competency, the student should be able to:
6.1 Define and determine the following for square matrices: eigenvalues, eigenvectors, characteristic polynomials and characteristic equations and prove basic properties about them
6.2 Diagonalize an n x n matrix when it has n linearly independent eigenvectors

Competency 7: Linear Transformations
Behavioral Objectives: In order to attain this competency, the student should be able to:
7.1 Recognize, define and give examples of linear transformations from one real vector space to another including linear transformations from R^n to R^m
7.2 Define and determine the kernel and range of a linear transformation
7.3 Prove basic properties about linear transformations
7.4 Represent linear transformations in matrix form
7.5 Determine the inverse of a linear transformation when it is one-to-one and onto
7.6 Use linear transformations to prove that two vector spaces are isomorphic
7.7 Define and prove basic results about similar matrices

Appendix E: Competencies for Preparation in Differential Equations

Competency 1: Basic Concepts
Behavioral Objectives: To obtain this competency, students should be able to

1.1 Classify differential equations as ordinary and partial; classify ordinary differential equations (ODEs) by linearity and by order
1.2 Understand the concept of solutions and verify solutions by substitution
1.3 Explain the difference of the general solutions of ODEs and the solutions to initial value problems (IVPs).
1.4 Understand the concepts of direction fields associated with first order ODEs and integral curves. Sketch the direction fields and some typical integral curves for such ODEs

Competency 2: First Order Ordinary Differential Equations
Behavioral Objectives: To obtain this competency, students should be able to

2.1 Recognize various types of first order ODEs and find their general solutions by elementary integration. Find the solution of an IVP of such equations by determining the appropriate constant. Such equations should include linear first order and separable ODEs, Bernoulli equations, homogeneous equations, and exact equations. Identify simple integral factors for certain non-exact equations. Develop a sense of solving an ODE using appropriate substitutions
2.2 Determine if a first order linear IVP has a unique solution and the interval of existence
2.3 Understand the result on existence/uniqueness/interval of existence of solutions of an IVP of a general first order ODE. Recognize the differences between linear and nonlinear ODEs.
2.4 Set up and solve IVPs for various applied problems such as those involving exponential growth/decay, mixing, and mechanics/physics.
2.5 Understand the logistic model in population dynamics. Understand stability properties of equilibrium solutions of first order autonomous equations and determine their stability properties using linear stability criteria.

Competency 3: Second Order Ordinary Differential Equations
Behavioral Objectives: To obtain this competency, students should be able to

3.1 Explain the principle of superposition and the relation between the general solutions of a second order linear homogeneous ODE and the corresponding nonhomogeneous one
3.2 Understand the basic theory of second order linear homogeneous ODEs: linear dependence and independence, the Wronskian, the fundamental set of solutions; Abel's theorem and its consequences
3.3 Find the general solutions of second order linear homogeneous ODEs with constant coefficient using the characteristic equations.
3.4 Explain and use the technique of reduction of order to find the general solution of a linear homogeneous ODE if a nontrivial solution is known.
3.5 Explain and apply the method of undetermined coefficients to appropriate second order linear nonhomogeneous ODEs to find particular solutions and the general solutions
3.6 Explain and apply the method of variation of parameters to find a particular solution of a second order linear nonhomogeneous ODE if the general solution of the corresponding homogeneous equation is known.
3.7 Solve appropriate applied problems related to mechanical or electric oscillations.
Explain the terms: free oscillations, forcing, damping, resonance, etc.
3.8 Find the solution to an IVP for any second order linear ODEs from the general solution

Competency 4: Series Solutions
Behavioral Objectives: To obtain this competency, students should be able to

4.1 Determine whether a given point is an ordinary point, a regular singular point, or an irregular singular point for any given second order linear homogeneous ODE
4.2 Find the two linearly independent series solutions, or at least the first several terms in each near an ordinary point. Determine the minimum radius of convergence of these series solutions from the coefficients of the ODE.
4.3 Recognize Euler ODEs and find their general solutions
4.4 Find the series solutions or at least the first several terms of two linearly independent series solutions of a second order ODE near a regular singular point by Frobenius method under appropriate condition on the exponents of singularity

Competency 5: Laplace Transforms
Behavioral ObjectivesT

5.1 Understand the definition of the Laplace transform, calculate the Laplace transforms of simple functions, and determine whether the Laplace transform of a given function exists

5.2 Use tables and general properties (linearity, derivative, translation) of Laplace transform to find the Laplace transform or the inverse Laplace transform of a given function.

5.3 Use Laplace transforms to solve a nonhomogeneous second-order IVP, where the forcing function could be discontinuous (expressed in terms of unit step functions), or periodic, or involving impulse functions.

5.4 Understand the unit impulse function, its Laplace transform and applications in the context of ODEs

5.5 Define the convolution of two functions, calculate it, and understand the convolution theorem

Pennsylvania Department of Education
Transfer and Articulation Oversight Committee
Approved by TAOC on November 22, 2010
Amended April 11, 2012

Appendix F: Transfer Credit Framework

Students who successfully complete courses from the approved categories below can have their credits transferred and counted towards graduation at any of the participating PA TRAC colleges and universities. Please be aware that certain majors may have specific requirements prescribed by external agencies. It is the student's responsibility to work with an advisor to select appropriate courses as they relate to the major.

Category 1 (Select 1 course)	**Category 2** (Select 1 course)	**Category 3** (Select no more than 2 courses)	**Category 4** Must include lab (Select no more than 2 courses)	**Category 5** (Select no more than 2 courses)	**Category 6** (Select no more than 2 courses)
English Composition	Public Speaking	Calculus I	General Chemistry I (majors & non-majors courses)	General Psychology	Introduction to Music
		Precalculus	General Chemistry II (majors & non-majors courses)	Introduction to Sociology	Introduction to Philosophy
		Elementary Statistics	General Biology I (majors & non-majors courses)	American National Government	Elementary Spanish I
		College Algebra	General Biology II (majors & non-majors courses)	Educational Psychology	Elementary Spanish II
		Foundations of Mathematics	General Physics I (non-calculus)	History of Western Civilization II	Painting I
			General Physics II (non-calculus)	Principles of Macroeconomics	Elementary French I
			Anatomy & Physiology I	Principles of Microeconomics	Elementary French II
			Anatomy & Physiology II	U.S. History I	Drawing I
			Introduction to Astronomy	U.S. History II	Ethics
				History of Western Civilization I	Introduction to Art
				Contemporary Social Problems	German I
				Introduction to Anthropology	German II
				Human Growth & Development	Introduction to Literature can also be known as Introduction to Poetry, Interpreting Literature, Reading Literature, Theses in Literature, Topics in Literature, Current Themes in Literature
				Child Psychology	Survey of American Literature
					Literature of the Western World
					World Literature
					American Literature
					Survey of English Literature
					Introduction to Theatre

Pennsylvania Department of Education
Transfer and Articulation Oversight Committee
Approved by TAOC on November 22, 2010
Amended April 11, 2012

ADDENDUM
GENERAL STATEWIDE PROGRAM-TO-PROGRAM ARTICULATION in PENNSYLVANIA
(Revised April 11, 2012)

WHEREAS, the General Assembly of the Commonwealth of Pennsylvania enacted Act 114 of 2006, which added to the Public School Code of 1949, Article XX-C entitled "Transfers of Credits Between Institutions of Higher Education" (referred to in this Agreement as the "Statewide Transfer System");

WHEREAS, Act 114 of 2006 requires all community colleges in Pennsylvania and Pennsylvania State System of Higher Education (PASSHE) universities to participate in the Statewide Transfer System;

WHEREAS, Act 114 of 2006 permits independent and state-related institutions of higher education in Pennsylvania, as each is defined in Article XX-C, to elect to participate in the Statewide Transfer System;

WHEREAS, the General Assembly of the Commonwealth of Pennsylvania enacted Act 50 of 2009, which requires institutions participating in the Statewide Transfer System to accept the transfer of Associate of Arts and Associate Science degrees into parallel baccalaureate programs and recognize all competencies attained within the associate degree program;

WHEREAS, Act 50 of 2009 defines an Associate of Arts (AA) or Associate of Science (AS) degree containing a minimum of 60 college-level credits and designed primarily for transfer to a baccalaureate institution;

WHEREAS, Act 50 of 2009 requires the Transfer Articulation Oversight Committee (TAOC), as established in section 2004-C of the Public School Code of 1949, to identify Associate of Arts and Associate of Science degree programs for transfer with full junior standing into parallel baccalaureate degrees annually; and,

WHEREAS, Act 50 of 2009 requires members of the Transfer Articulation Oversight Committee established in section 2004-C of the Public School Code of 1949, to identify modifications that may be required in existing associate or baccalaureate degrees to satisfy external accreditation or licensure requirement;

All Institutions participating in the Statewide Transfer System enter into this Articulation Agreement and mutually agree as follows:

1. The statewide program-to-program articulation agreement ensures that students who complete an AA or AS degree from a participating institution will have their coursework and credits transfer into a parallel baccalaureate program with full junior standing and without the need for course-by-course equivalency.

2. Students are subject to the admissions and transfer credit policies of the participating institutions. The admissions and transfer credit policies for all of the institutions participating in Pennsylvania's college credit transfer system can be found at www.PAcollegetransfer.com.

3. The AA or AS degree must include a minimum of 60 college-level credits designed and acceptable for transfer, not including developmental or remedial courses or career, technical or applied courses.

4. The transfer of coursework with a grade less than a C (2.0 on a 4.0 scale) in the AA or AS will be consistent with the policies of native students at the participating college or university.

5. Students and institutional personnel will be able to find out which institutions offer articulated programs by accessing a searchable database located at www.PAcollegetransfer.com. PDE will maintain this database through program information provided to TAOC by the individual participating institutions.

6. References to courses in all agreements designate competencies and are not to be construed as making a reference to a specific course at a specific institution. Course titles in the agreements are presented for guidance in advising students

as to which coursework they should take even though the course at the student's college may not have the specific title mentioned in the agreement.[2]

7. Responsibilities of Associate Degree Institutions

a. The AA or AS degree leading to a parallel bachelor degree will include the minimum number of credits and competencies of major-specific coursework as defined by the Agreement.

b. Any remaining AA or AS degree requirements will be accepted from arts and sciences electives designed and acceptable for transfer, not including developmental, remedial, career, technical or applied courses.

c. By awarding the AA or AS, the Associate Degree Institution is validating that the student has met the competency requirements outlined in the Agreement.

8. Responsibilities of Bachelor Degree Institutions

a. The Bachelor Degree Institution will recognize all competencies attained within the AA or AS degree and accept a transfer student who has earned the associate degree with full junior standing into a parallel baccalaureate degree program.

b. All decisions made with respect to the transfer process shall be based on the principle of equivalence of expectations and requirements for native and transfer students.

c. A transfer student's admission into the parallel baccalaureate degree will be subject to the Bachelor Degree Institution's specific requirements for admission to that major and be consistent with such requirements for native students.

9. Agreement Revision and Assessment

a. Once a statewide program-to-program articulation agreement has been approved by TAOC, no amendments to the agreement can be offered by any party within the initial six (6) months of the agreement. After that time, a TAOC member with a proposed amendment to an approved agreement should submit the change to PDE.

Amendments that are offered as clarifying or technical but do not alter the substantive portions or intent of the agreement must be forwarded to TAOC. TAOC representatives will have at least thirty (30) days to review, comment and approve or deny the proposed amendments.

Amendments that seek to alter the substantive nature or intent of the agreement in any part must be forwarded to the appropriate PAC for review and consideration. The PAC will then make a recommendation to the TAOC, and TAOC shall approve or deny the proposed amendments.[3]

b. PDE and TAOC will exercise responsibility for monitoring the effectiveness of the Agreement and its implementation.

c. PDE shall collect data annually from the participating institutions that will enable the Department and TAOC to assess the effectiveness of the implementation of the Agreement in fostering a seamless transfer process and the academic success of transfer students at the senior institutions.

10. Transfer Appeal Process

a. In accordance with Pennsylvania's Statewide Transfer System, each Bachelor Degree Institution shall have a procedure through which a transfer student can appeal a decision that he/she believes is not consistent with this Agreement.

b. The Transfer Appeal Process shall be published, at minimum, in the institution's catalog and posted to the Commonwealth's official website of the Statewide Transfer System, www.PAcollegetransfer.com.

[2] Adopted by TAOC and added to the agreement on April 11, 2012.
[3] Approved by TAOC and added to agreement on August 18, 2011.

11. **Institutional Resolution of Disputes**
    a. In the event that an Associate Degree Institution considers the decision of a Bachelor Degree Institution to be inconsistent with this Agreement, the Associate Degree Institution shall consult directly with the Bachelor Degree Institution and attempt to resolve the matter.

    b. If the institutions are unable to resolve the issue, the Associate Degree Institution may submit their concern to PDE for consideration by the TAOC Dispute Resolution Committee. The Dispute Resolution Subcommittee will act according to the policies and procedures developed by TAOC as part of the Statewide Transfer System. The determination made by the Dispute Resolution Subcommittee will be binding upon the parties.

12. **Implementation Date and Applicability**
    Having fulfilled the requirements outlined in the Program-to-Program Articulation Agreement, students transferring with an AA or AS degree from a participating institution will be considered by the receiving baccalaureate degree institution to have received adequate preparation in the field of study at the foundation level and therefore eligible to transfer as a junior into advanced major coursework.

    Participating institutions will enact the Agreement in accordance to the timeline outlined by the TAOC, but no later Fall 2013.[4]

    Continuation of the agreement remains in effect until such time as all cooperating institutions of the Statewide Transfer System formally approve any revisions.

GLOSSARY OF TERMS

Articulation: The aligning of curriculum between institutions of higher education to ensure the efficient and effective movement of students among those institutions.

Associate of Arts (AA) and Associate of Science (AS) Degree: A degree consisting of at least 60 college-level credits and designed for transfer into a baccalaureate degree program.

Foundation Coursework: Courses at a level of comprehension usually associated with freshman and sophomore students and typically offered during the first half of a baccalaureate degree program. Such coursework typically does not have course prerequisites.

Native Student: A student who entered a given college or university without first matriculating at another college.

Parallel Baccalaureate Degree: A bachelor degree program in a comparable field of study and with similar foundation-level major-specific competencies as an associate degree program.

Receiving Institution: The college or university where a transfer student plans to enroll and to apply previously earned credit toward a degree program.

Transfer Credit: The credit granted by a college or university for college-level courses or other academic work completed at another institution.

Transfer Student: A student who enters a participating college or university after earning college-level credit at another college or university.

Transfer: The process by which a student moves from one postsecondary institution to another. Also refers to the mechanics of credit, course and curriculum exchange between institutions.

[4] Agreements approved by TAOC prior to August 31, 2011 must be implemented by the institutions by Fall 2012. Agreements approved by TAOC after August 31, 2011 but before May 1, 2012 must be implemented by the institutions by Fall 2013.

Advanced Coursework: Courses with advanced depth of content knowledge in the field of study and carry the expectation of more complex competencies identified in the expected student learning outcomes is referred to as advanced coursework. These courses often have prerequisites and are usually beyond the "Introduction to…" or "Foundation of…" level.